RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4755

Further Concepts For Advanced Mathematics (FP1)

Friday **21 JANUARY 2005** Afternoon 1 hour 30 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 72.

This question paper consists of 4 printed pages.

Section A (36 marks)

1 You are given the matrix $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ -2 & 1 \end{pmatrix}$.

Find the inverse of **M**.

The transformation associated with **M** is applied to a figure of area 2 square units. What is the area of the transformed figure? [3]

2 **(i)** Show that $\dfrac{1}{r+1} - \dfrac{1}{r+2} = \dfrac{1}{(r+1)(r+2)}$. [2]

(ii) Hence use the method of differences to find the sum of the series

$$\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)}.$$ [4]

3 **(i)** Solve the equation $\dfrac{1}{x+2} = 3x+4$. [3]

(ii) Solve the inequality $\dfrac{1}{x+2} \leqslant 3x+4$. [4]

4 Find $\sum_{r=1}^{n} r^2(r+2)$, giving your answer in a factorised form. [6]

5 The roots of the cubic equation $x^3 + 2x^2 + x - 3 = 0$ are α, β and γ.

Find the cubic equation whose roots are $\alpha + 1, \beta + 1$ and $\gamma + 1$, simplifying your answer as far as you can. [6]

6 Prove by induction that $\sum_{r=1}^{n} r2^{r-1} = 1 + (n-1)2^n$. [8]

Section B (36 marks)

7 A curve has equation $y = \dfrac{(2x-3)(x+1)}{(x+4)(x-2)}$.

(i) Write down the values of x for which $y = 0$. [1]

(ii) Write down the equations of the three asymptotes. [3]

(iii) Determine whether the curve approaches the horizontal asymptote from above or from below for

(*A*) large positive values of x,

(*B*) large negative values of x. [3]

(iv) Sketch the curve. [3]

(v) Solve the inequality $\dfrac{(2x-3)(x+1)}{(x+4)(x-2)} \leqslant 2$. [4]

8 Two complex numbers are given by $\alpha = 2 - \mathrm{j}$ and $\beta = -1 + 2\mathrm{j}$.

(i) Find $\alpha + \beta$, $\alpha\beta$ and $\dfrac{\alpha}{\beta}$ in the form $a + b\mathrm{j}$, showing your working. [6]

(ii) Find the modulus of α, leaving your answer in surd form. Find also the argument of α. [2]

(iii) Sketch the locus $|z - \alpha| = 2$ on an Argand diagram. [2]

(iv) On a separate Argand diagram, sketch the locus $\arg(z - \beta) = \frac{1}{4}\pi$. [2]

[Turn over

9 You are given the matrix $\mathbf{M} = \begin{pmatrix} 0.8 & 0.6 \\ 0.6 & -0.8 \end{pmatrix}$.

(i) Calculate $\mathbf{M}^2$. [1]

You are now given that the matrix **M** represents a reflection in a line through the origin.

(ii) Explain how your answer to part **(i)** relates to this information. [1]

(iii) By investigating the invariant points of the reflection, find the equation of the mirror line. [3]

(iv) Describe fully the transformation represented by the matrix $\mathbf{P} = \begin{pmatrix} 0.8 & -0.6 \\ 0.6 & 0.8 \end{pmatrix}$. [2]

(v) A composite transformation is formed by the transformation represented by **P** followed by the transformation represented by **M**. Find the single matrix that represents this composite transformation. [2]

(vi) The composite transformation described in part **(v)** is equivalent to a single reflection. What is the equation of the mirror line of this reflection? [1]

Mark Scheme

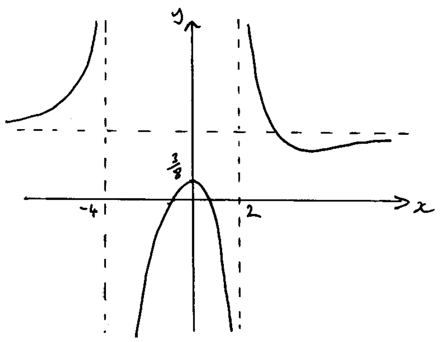
Qu	Answer	Mark	Comment
Section A			
1	Det **M** = 8 $\mathbf{M}^{-1} = \frac{1}{8}\begin{pmatrix} 1 & -3 \\ 2 & 2 \end{pmatrix}$ Area = 16 square units	B1 B1 B1 **[3]**	
2(i)	$\frac{1}{r+1} - \frac{1}{r+2} \equiv \frac{(r+2)-(r+1)}{(r+1)(r+2)} \equiv \frac{1}{(r+1)(r+2)}$	M1 A1 **[2]**	
2(ii)	$\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)} = \sum_{r=1}^{n} \left[\frac{1}{(r+1)} - \frac{1}{(r+2)} \right]$ $= \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) + \left(\frac{1}{4}-\frac{1}{5}\right) + \ldots\ldots\ldots$ $+\left(\frac{1}{n}-\frac{1}{n+1}\right) + \left(\frac{1}{n+1}-\frac{1}{n+2}\right)$ $= \frac{1}{2} - \frac{1}{n+2}$	M1 M1 M1 A1 . **[4]**	 First two terms in full. Last two terms in full. Give B4 for correct without working.
3(i)	$3x^2 + 10x + 7 = 0$ $x = \frac{-7}{3}$ or $x = -1$	M1 A1, A1 **[3]**	Quadratic from equation 1 mark for each solution.
3(ii)	Graph sketch or valid algebraic method. $-2 > x \geq -\frac{7}{3}$ or $x \geq -1$	G2 or M2 A1 A1 **[4]**	Sketch with $y = \frac{1}{x+2}$ and $y = 3x + 4$ or algebra to derive $0 \leq \frac{(3x+7)(x+1)}{(x+2)}$ with consideration of behaviour near critical values of *x*. Lose one mark if incorrect inequality symbol used in $-2 > x \geq -\frac{7}{3}$ <u>Alternative</u> For sensible attempt but vertical asymptote at *x* = -2 not considered, give M1 only.

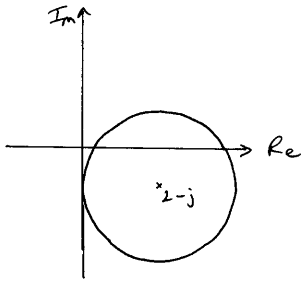
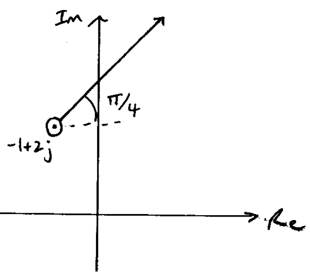
Qu	**Answer**	**Mark**	**Comment**
4	$\sum_{r=1}^{n} r^2(r+2) = \sum_{r=1}^{n} r^3 + 2\sum_{r=1}^{n} r^2$ $= \frac{1}{4}n^2(n+1)^2 + \frac{1}{3}n(n+1)(2n+1)$ $= \frac{1}{12}n(n+1)[3n(n+1)+4(2n+1)]$ $= \frac{1}{12}n(n+1)(3n^2+11n+4)$ i.s.w.	M1,A1 M1,A1 M1 A1 **[6]**	Separate sums Use of formulae. Follow through from incorrect expansion in line 1. Factorising
5	$w = x+1 \Rightarrow x = w-1$ $\Rightarrow (w-1)^3 + 2(w-1)^2 + (w-1) - 3 = 0$ $\Rightarrow w^3 - 3w^2 + 3w - 1 + 2w^2 - 4w + 2 + w - 1 - 3 = 0$ $\Rightarrow w^3 - w^2 - 3 = 0$	B1 M1 A1, A1,A1 A1 **[6]**	Substitution. For substitution $w = x$-1 give B0 but then follow through. Substitute into cubic Expansion Simplifying

Qu	Answer	Mark	Comment
5	Alternative $a+b+g=-2$ $ab+bg+ag=1$ $abg=3$ Coefficients: $w^2=-1$ $w=0$ constant $=-3$ Correct final cubic expression $w^3-w^2-3=0$	M1 A1 B1 B1 B1 B1 **[6]**	Attempt to calculate these All correct
Qu	**Answer**	**Mark**	**Comment**
6	For $k=1$, $1\times 2^{1-1}$ and $1+(1-1)2^1=1$, so true for $k=1$	B1	
	Assume true for $n=k$	E1	Explicit statement: 'assume true for $n=k$' Ignore irrelevant work
	Next term is $(k+1)2^{k+1-1}=(k+1)2^k$	M1 A1	Attempt to find (k +1)th term Correct
	Add to both sides RHS $=1+(k-1)2^k+(k+1)2^k$	M1	Add to both sides
		A1	Correct simplification of RHS

	$= 1 + (k-1)2^k + (k+1)2^k$ $= 1 + 2^k(k-1+k+1)$ $= 1 + 2^k \times 2k$ $= 1 + 2^{k+1}k$ $= 1 + ((k+1)-1)2^{k+1}$ But this is the given result with $(k+1)$ replacing *k*. Therefore if it is true for *k* it is true for $(k+1)$. Since it is true for $k = 1$ it is true for *k* = 1, 2, 3, … .	E1 E1 **[8]**	Statement 'if true for *k*, true for *k* + 1', or equivalent (only give if simplification valid) Relating to *k* = 1, accept 'by induction'
			Section A Total: 36

Section B

7(i)	$x=\frac{3}{2}$ and -1	B1 **[1]**	Both.
7(ii)	$x=2,\ x=-4$ and $y=2$	B1, B1,B1 **[3]**	
7(iii)	Large positive x, $y \to 2^-$ (e.g. consider $x=100$) Large negative x, $y \to 2^+$ (e.g. consider $x=-100$)	B1 B1 B1 **[3]**	Evidence of method needed for this mark.
7(iv)	Curve 3 branches Asymptotes marked Correctly located and no extra intercepts 	B1 B1 B1 **[3]**	Consistent with their (iii)
7(v)	$y=2 \Rightarrow 2=\dfrac{(2x-3)(x+1)}{(x+4)(x-2)}$ $\Rightarrow 2x^2+4x-16=2x^2-x-3$ $\Rightarrow x=\dfrac{13}{5}$ From sketch, $y \le 2$ for $x \ge \dfrac{13}{5}$ or $2>x>-4$	M1 A1 A1, B1 **[4]**	Some attempt at rearrangement May be given retrospectively B1 for $2>x>-4$

8(i)	$a+b=1+\mathrm{j}$ $ab=(2-\mathrm{j})(-1+2\mathrm{j})$ $=-2+4\mathrm{j}+\mathrm{j}-2\mathrm{j}^2$ $=5\mathrm{j}$ $\dfrac{a}{b}=\dfrac{(2-\mathrm{j})(-1-2\mathrm{j})}{(-1+2\mathrm{j})(-1-2\mathrm{j})}=\dfrac{-2-4\mathrm{j}+\mathrm{j}+2\mathrm{j}^2}{5}$ $=\dfrac{-4}{5}-\dfrac{3}{5}\mathrm{j}$ or $\dfrac{-4-3\mathrm{j}}{5}$	B1 M1 A1 M1, A1 A1 **[6]**	. Use of conjugate of denominator
8(ii)	$r=\lvert a\rvert=\sqrt{5}$ $q=\arg a=-0.464$	B1 B1 **[2]**	Accept degree equivalent (-26.6°)
8(iii)	Circle, centre $2-\mathrm{j}$, radius 2 	B1 B1 **[2]**	Argand diagram with circle. 1 mark for centre, one mark for radius.
8(iv)	Half line from $-1+2\mathrm{j}$, making an angle of $\dfrac{\pi}{4}$ to the positive real axis. 	B1 B1 **[2]**	Argand diagram with half line. One mark for angle.

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	$\mathbf{M}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}$	B1 **[1]**	.
9(ii)	$\mathbf{M}^2$ gives the identity because a reflection, followed by a second reflection in the same mirror line will get you back where you started OR reflection matrices are self-inverse.	E1 **[1]**	
9(iii)	$\begin{pmatrix} 0.8 & 0.6 \\ 0.8 & -0.6 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\Rightarrow 0.8x + 0.6y = x$ and $0.6x - 0.8y = y$ Both of these lead to $y = \frac{1}{3}x$ as the equation of the mirror line.	M1 A1 A1 **[3]**	Give both marks for either equation or for a correct geometrical argument
9(iv)	Rotation, centre origin, 36.9° anticlockwise.	B1, B1 **[2]**	One for rotation and centre, one for angle and sense. Accept 323.1° clockwise or radian equivalents (0.644 or 5.64).
9(v)	$\mathbf{MP} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	M1, A1 **[2]**	
9(vi)	$y = 0$	B1 **[1]**	
			Section B Total: 36
			Total: 72

Examiner’s Report

4755: Further Concepts For Advanced Mathematics (FP1)

General Comments

The paper proved very appropriate for its candidature.

All questions were accessible and most candidates were able to make some attempt at all of them. The general standard of work was high. Many questions tested the use of algebra and it was encouraging to see a generally high standard of manipulation. The solution of inequalities was the most consistent area of difficulty.

The last question was generally the least well answered, which indicates that some candidates did experience problems in completing the paper in the time available. Also, a small number of candidates seem not to have seen the instruction to turn over and so did not attempt the last question.

Comments on Individual Questions

1) **Matrices**

The question was generally well answered. However, a few candidates seemed unaware of the relationship between the determinant of a transformation matrix and the area scale factor of the transformation it represents.

2) **Method of differences**

This question was well answered. Almost all candidates were able to do the first part, involving algebraic fractions. Most went on to complete the second part, but a few seemed unfamiliar with the method of differences and/or ignored the 'hence' and so got themselves into a mess.

3) **Equation/inequality**

(i) This part reduced to a quadratic equation and most candidates got it right.

(ii) This involved a related inequality and only the best candidates were successful. Most who did get it right drew a graph. There were a few correct solutions based on algebra alone, but most failed to consider the behaviour either side of the vertical asymptote at $x = 2$. The weaker candidates often scored no marks for this part of the question.

4) **Series summation**

This question was well answered and many candidates got it fully right. The most common mistake was failure to open up the brackets in the summation, often followed by an attempt to multiply two series sums together.

5) **Roots of a cubic equation**

This was generally well answered but few candidates used the elegant (and short) method of using a substitution, replacing x by $w - 1$. Most opted for a solution involving sums and products of roots, which was both longer and more prone to error.

6) **Proof by induction**

This was well answered by the best candidates but others had difficulty. Common faults were a failure to set out the argument logically and getting tied up in the algebra when establishing the result for $k + 1$ terms. The weakest candidates often did not get beyond showing that the conjecture was true for $n = 1$.

7) **Graph sketching**

Virtually all candidates were able to obtain some marks on this question but only the very best achieved full marks.

(i) The vast majority of candidates answered this correctly.

(ii) Nearly all candidates wrote down the correct equations for the vertical asymptotes. Weaker candidates either omitted to give a horizontal asymptote, or said it was $y = 0$.

(iii) Many candidates showed no workings and so could only achieve two of the three available marks. Many candidates failed to appreciate that the graph approached the horizontal asymptote from below for large positive values of x and this then resulted in a failure to solve the inequality fully in part (v).

(iv) Most candidates earned one or two marks for this. However, a significant number of those who realised the graph approached the horizontal asymptote from below for large positive values of x then drew the right-hand branch of their graph upside-down. They were perhaps under the misapprehension that a graph may not cross any of its asymptotes.

(v) Only the very best candidates got this fully correct, though quite a large number spotted that the region between the vertical asymptotes satisfied the inequality.

8) **Complex numbers**

This question was generally well answered and many candidates obtained full marks.

(i) Almost all candidates were able to add and multiply two complex numbers correctly but a few of the weakest had difficulties dividing two complex numbers.

(ii) Nearly all candidates could calculate the modulus of a complex number correctly. Many gave an incorrect sign for the argument (the equivalent reflex angle was accepted as correct). A quick sketch of the complex number on an Argand diagram is an easy way for candidates to avoid this error.

(iii) Most candidates got this right, though a centre at (2, 1), rather than (2, -1) was a common error.

(iv) Only the best candidates got this right. Many gave a circle rather than a half line. Another common error was to start the half-line from the origin.

9) **Matrices**

While there were many good answers to this question, there were also many that were distinctly sketchy. Some candidates may have been under time-pressure. A small number may not have attempted the question if they missed the instruction to turn over.

(i) The vast majority of candidates answered this correctly.

(ii) Many candidates failed to explain this adequately, with a significant number stating that if $\mathbf{M}^2 = \mathbf{I}$, $\mathbf{M}$ must represent a reflection, which is incorrect.

(iii) Only the stronger candidates answered this correctly. A few used geometry, rather than direct matrix work to find a solution, which was perfectly acceptable.

(iv) The better candidates earned at least one mark for this, though many lost a mark by not stating the origin as the centre of rotation. Several candidates assumed $\mathbf{P}$ must represent a reflection.

(v) Quite a large number of candidates multiplied $\mathbf{P}$ and $\mathbf{M}$ in the wrong order.

(vi) Most who answered (v) correctly also answered this correctly.